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MATHEMATICS 23–33

A. PROGRAM RATIONALE AND PHILOSOPHY

To set goals and make informed choices, students need an array of thinking and problem-solving skills. Fundamental to this is an understanding of mathematical techniques and processes that will enable them to apply the basic skills necessary to address everyday mathematical situations, as well as acquire higher order skills in logical analysis and methods for making valid inferences.

A knowledge of mathematics is essential for a well-educated citizenry. However, the need for and use of mathematics in the life of the average citizen is changing. **Emphasis has shifted from the memorization of mathematical formulas and algorithms toward a more dynamic view of mathematics as a precise language, used to reason, interpret and explore.** There is still a need for the logical development of concepts and skills as a basis for the appropriate use of mathematical information to solve problems. The more traditional problem-solving techniques, combined with techniques such as estimation and simulation, and incorporated with modern technology, are the tools with which mathematical problems are solved.

Change in the way in which mathematics is used is necessitating a concurrent change in the emphases of mathematics education. Students need an expanded list of fundamental concepts but will also need to understand the ideas that make up those concepts and how they are related. They also require familiarity with their applications. Most important, students must be able to solve problems using the mathematical processes developed, and be confident in their ability to apply known mathematical skills and concepts in the acquisition of new mathematical knowledge. In addition, the ability of technology to provide quick and accurate computation and manipulation, to enhance conceptual understanding and to facilitate higher order thinking, should be recognized and used by students.

The majority of students who enter senior high school exhibit mainly concrete operational behaviours with regard to mathematics. Students are expected to acquire much abstract understanding in senior high school mathematics courses. The course content of the Senior High School Mathematics Program is cognitively appropriate for the students and should be presented in a way that is consistent with the students' ability to understand.

The Senior High School Mathematics Program includes the course sequences Mathematics 16–26, 14–24, 23–33, plus Mathematics 31. Transfer by students among courses of different sequences is possible. The course sequences commensurate with differing abilities, interests and aspirations, are designed to enable students to have success in mathematics. As well, the mathematics program reflects the changing needs

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of society, and provides students with the mathematical concepts, skills and attitudes necessary to cope with the challenges of the future.

The Mathematics 23–33 sequence is the mainstream mathematics program designed for students who require mathematics to prepare them for many post-secondary programs at universities, colleges, trades and employment. This sequence emphasizes an inductive development of topics in algebra, geometry, trigonometry, statistics and consumer mathematics. Mathematics 23–33 satisfies the requirements for the Alberta High School Diploma.

B. GENERAL LEARNER EXPECTATIONS

Students are expected to be mathematically literate at the conclusion of their senior high school mathematics education. **Mathematical literacy** refers to students' ability and inclination to manage the demands of their world through the use of mathematical concepts and procedures to communicate, reason and solve problems. More specifically, *students will be expected to:*

- have achieved understanding of the basic mathematical concepts, and developed the skills and attitudes needed to become responsible and contributing members of society
- apply basic mathematical skills and concepts in practical situations
- have developed the skills, concepts and attitudes that will ensure success in the mathematical situations that occur in future educational endeavours, employment and everyday life
- have developed the skills, concepts and attitudes that will enable the acquisition of mathematical knowledge beyond the conclusion of secondary education
- have developed critical and creative thinking skills
- be able to communicate mathematical ideas effectively
- understand how mathematics can be used to investigate, interpret and make decisions in human affairs
- understand how mathematics can be used in the analysis of natural phenomena
- understand the connections and interplay among various mathematical concepts and between mathematics and other disciplines
- understand and appreciate the positive contributions of mathematics, as a science and as an art, to civilization and culture.

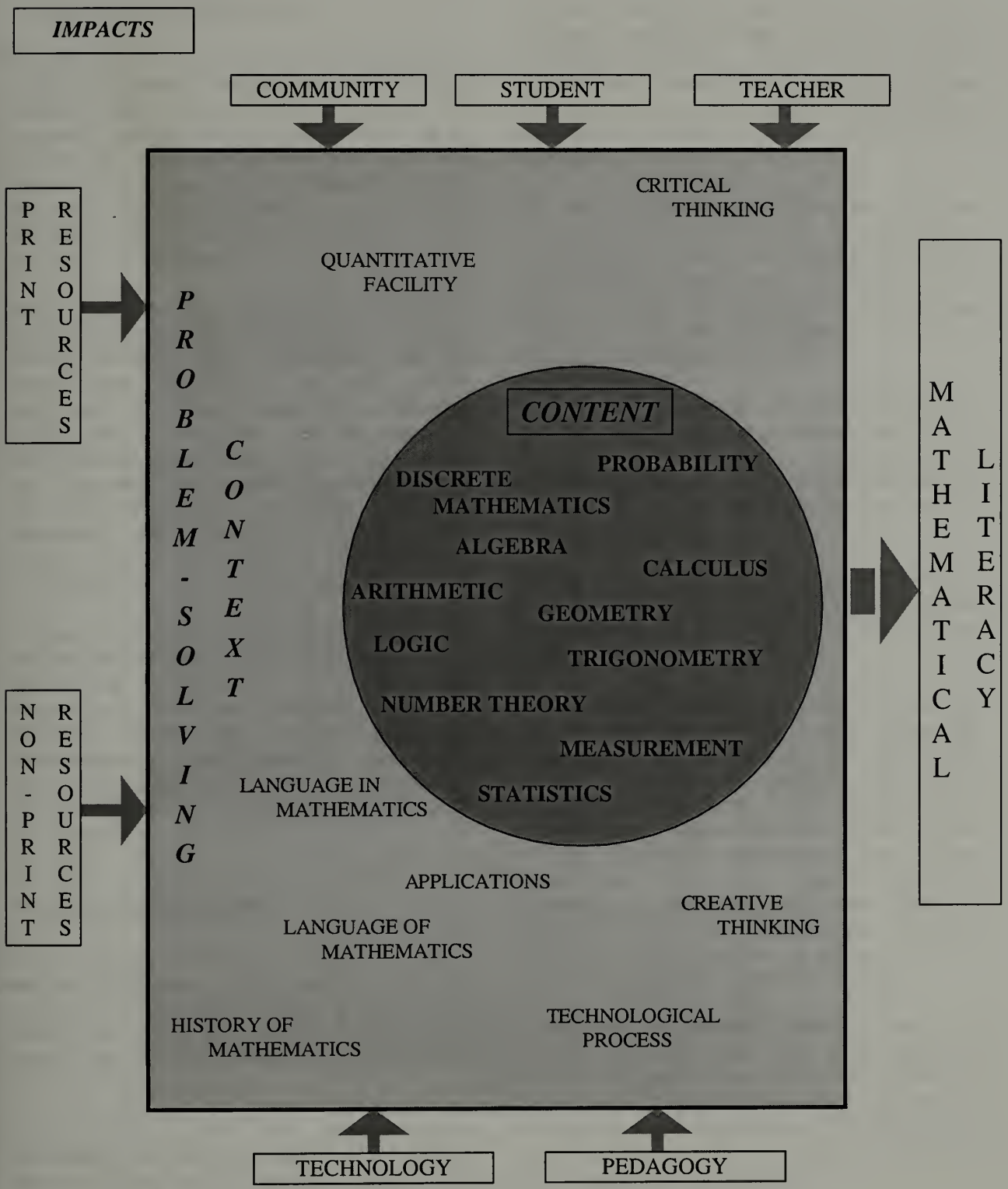
A General Model for Mathematical Literacy for Senior High School Programs, which outlines the factors that affect what and how students learn as they become mathematically literate, is presented on the following page. The model is fluid in that the content can be learned in a problem-solving context that engages any of a number of vehicles as the learning focus. At the same time, the students involved in the learning situation are affected in what and how they learn by forces that have impact on them.

The **Content** of the Senior High School Mathematics Program is the body of knowledge that is to be acquired by students. In the various senior high school mathematics courses, it is made up of topics that can be categorized into one or more of the listed strands.

In each course in the Senior High School Mathematics Program, students will focus on problem solving. The **Problem-solving Context** refers to the instruction emphases within which the specific content expectations can be acquired. The various entries indicated in the model suggest processes that belong to the problem-solving context and may be used by students as vehicles for learning the content.

The **Impacts** on the problem-solving context are those skills, attitudes and experiences that are possessed by the students and teachers involved, as well as the resources they may use throughout the learning process. They include the influence exerted by the culture and beliefs of the community as reflected by the school. The effectiveness of the context in enabling a student to acquire the content is dependent on the skillful management by the teacher of those items that have an impact on a student's learning.

A General Model for Mathematical Literacy for Senior High School Programs



C. SPECIFIC LEARNER EXPECTATIONS

PROGRAM ORGANIZATION

The major part of the content of each senior high school mathematics course consists of topics required of all students who take the course. The **required content** comprises 80 per cent of the course and contains the concepts, skills and attitudes that all students are expected to acquire. As well, the required portion of all courses includes specific expectations for problem solving and the use of technology.

Each course includes a compulsory component comprising 20 per cent of the course, made up of **elective material** that is consistent with the content and expectations of the required component. **The elective material provides for enrichment, remediation, or innovative or experimental presentations or activities. It is not intended to provide acceleration or advanced placement.** However, horizontal enrichment and extension is appropriate and students should have access to elective material that serves their individual needs and interests.

Evaluation of students in the Senior High School Mathematics Program will involve assessment of the level of achievement of all **of the learner expectations, including concepts, skills and attitudes, as well as problem-solving and technological expectations.** For more information regarding evaluation, consult the *Teacher Resource Manual for Senior High Mathematics.*

PROGRAM STRUCTURE

At the beginning of each course is a list of **attitude** expectations. These attitudes embody a mathematical attitude or frame of mind for a student to view the world. The attitude expectations should be woven into the fabric of the entire course.

Following this are the **problem-solving** expectations that outline a variety of procedures, strategies, skills and checking techniques for solving problems. Because a major purpose for studying mathematics is to learn to solve problems, problem-solving expectations occur throughout all areas of the specific learner expectations. Students must have the background skills and knowledge necessary to achieve these expectations successfully, using problem-solving techniques.

The units of the course are broken into a number of concepts. Each unit begins with an **overview** of the concepts and skills included in the unit. This indicates reasons why the unit is being studied and how these particular concepts and skills fit into the development of major mathematical concepts.

A **concept** is an abstract or general idea about specific instances that have common properties or an identifiable relationship to one another. The concepts are presented as mathematical definitions or theorems or as statements of mathematical ideas or abstractions.

Supporting each concept are a number of skills. **Skills** are intellectual or physical capabilities that will be developed in the context of the particular concept.

Skills specifically related to the use of **technology** identify areas in which scientific calculators and/or computer technology are applied by students as tools to be used for calculations, manipulation or graphing, or to aid in the analysis of problems. Technological expectations are defined explicitly throughout the learner expectations. In many cases, a particular technology is indicated for investigation or analysis. It is in these situations that the use of technology enables students to engage in critical and creative thinking and problem solving.

Students will be expected to learn how and when to use technology and have a demonstrated proficiency in **estimation** and **mental arithmetic**. To use technology effectively, they must be able to judge the reasonableness of an answer and understand the importance of making a judgment about the result of a calculation.

The words verify and prove appear throughout the learner expectations. For the purposes of the Senior High School Mathematics Program, they are interpreted as:

- Verify – to substantiate the validity of an operation, solution, formula or theorem through the use of examples that may or may not be generalized;
- Prove – to substantiate the validity of an operation, solution, formula or theorem in general and to provide logical arguments for each step in the process.

Attitudes

1. *Students will be expected to* demonstrate an attitude associated with mathematical literacy. In particular, students will be expected to:

    1.1 be confident in their mathematical knowledge and in their ability to acquire new knowledge
    1.2 demonstrate persistence, resolve, flexibility and ingenuity in finding the solution to problems
    1.3 develop intellectual curiosity and openness to new ideas, insights and change in the pursuit of mathematical knowledge
    1.4 exhibit an attitude of curiosity and spontaneity, and appreciate creativity and innovation in representing situations mathematically
    1.5 be critical and constructive in approaching new ideas and new processes
    1.6 be aware of the importance of communication skills in mathematics
    1.7 appreciate the usefulness of computational competence, mathematical processes and problem-solving skills that are used in the decision-making and modelling processes in our society
    1.8 appreciate the contributions of mathematics to our culture and civilization.

Problem Solving

1. *Students will be expected to* demonstrate an understanding of the variety of procedures that can be used to understand problems. In particular, students will be expected to:

    1.1 read the problem thoroughly
    1.2 identify and clarify key components
    1.3 restate the problem, using familiar terms
    1.4 evaluate the given information as to sufficiency and relevancy
    1.5 interpret pictures, charts and graphs
    1.6 determine hidden assumptions
    1.7 ask relevant questions
    1.8 identify given, needed and wanted information
    1.9 diagram or model the problem situation
    1.10 use suitable notation
    1.11 determine valid inferences
    1.12 simulate a problem situation
    1.13 formulate situations into identifiable problems.

2. *Students will be expected to* develop a variety of strategies for use in the solution of mathematical problems. In particular, students will be expected to:

   2.1 conduct an investigation
   2.2 use estimation and approximation
   2.3 develop equations or use formulas
   2.4 use flow charts
   2.5 make lists and charts
   2.6 look for patterns
   2.7 work backward
   2.8 break the problem into smaller parts
   2.9 look for a simpler or related problem
   2.10 make diagrams or models
   2.11 use manipulatives
   2.12 choose and sequence a series of mathematical operations
   2.13 sketch the graph of a problem situation
   2.14 establish procedures to gather and organize data
   2.15 apply empirical or inductive processes
   2.16 use geometric construction and measurement techniques
   2.17 make and test a conjecture.

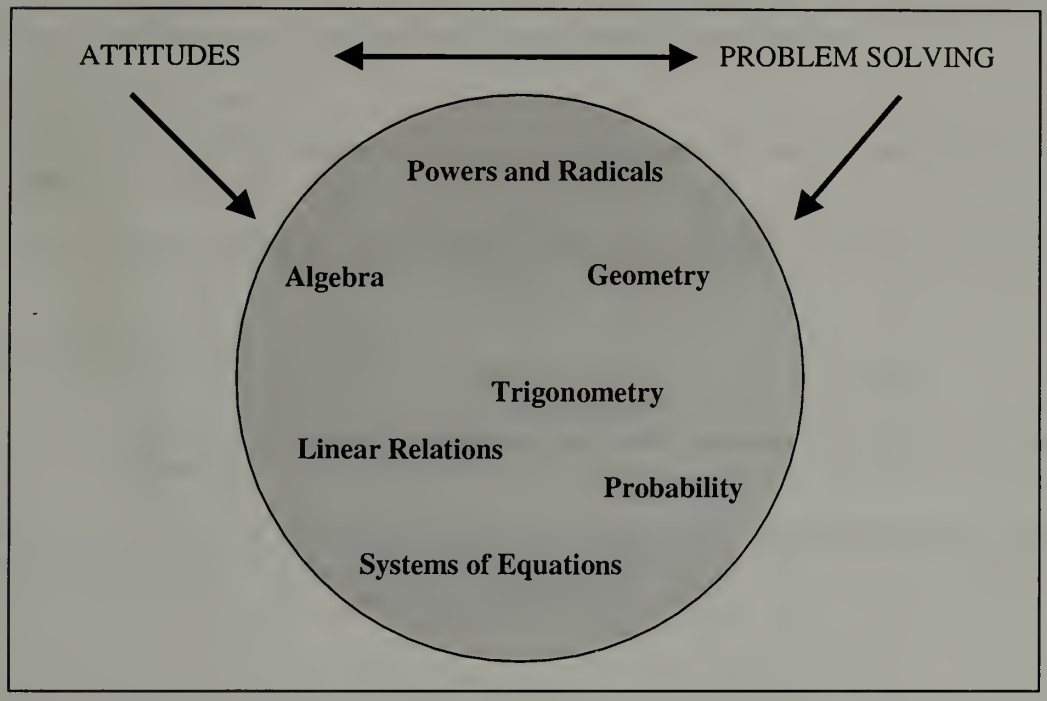
3. *Students will be expected to* develop a variety of skills that can be used to carry out the plan for the solution of a problem. In particular, students will be expected to:

   3.1 apply selected strategies
   3.2 present ideas clearly
   3.3 document the solution process
   3.4 use appropriate group behaviours
   3.5 use scientific graphing calculators and/or computers
   3.6 evaluate problem-solving strategies for effectiveness
   3.7 alter or abandon non-productive strategies
   3.8 search for additional information
   3.9 ask questions
   3.10 be open to inspirations, intuitions and "bright ideas".

4. *Students will be expected to* employ a variety of skills to help them look back over the solution of a problem. In particular, students will be expected to:

   4.1 determine the reasonableness of an answer
   4.2 explain the solution in oral or written form
   4.3 consider the possibility of additional solutions
   4.4 search for other strategies and processes of solution
   4.5 create and solve similar problems
   4.6 note the characteristics that will be identifiable in similar problems
   4.7 make a generalization
   4.8 examine the assumptions made and simplifications and modifications used for accuracy, effectiveness and efficiency.

MATHEMATICS 23 PROGRAM STRUCTURE



POWERS AND RADICALS

1. *Students will be expected to* demonstrate an understanding of the relationship between radicals and exponents.

   Students will be expected to:

   1.1 use the following terms as they occur in the study of radicals: index, radicand, radical sign, root

   1.2 transform expressions of the form $x^{a/b}$ to $\sqrt[b]{x^a}$ or $\left|\sqrt[b]{x}\right|^a$ and vice versa where x is a rational number, a is a natural number and $b = 2$

   1.3 calculate approximations for the square and cube roots of rational numbers

      1.3.1 find approximations for square and cube roots of rational numbers by using a calculator.

2. *Students will be expected to* demonstrate an understanding of how interest accumulates, if it is allowed to compound.

   Students will be expected to:

   2.1 calculate compound interest by using an iterative process
   2.2 calculate compound interest using the formula $A = P(1 + i)^n$, where A is the amount (principal and interest), i is the interest rate per conversion period and n is the number of periods

      2.2.1 use a calculator or spreadsheet in the calculation of compound interest
      2.2.2 solve problems involving the calculation of compound interest.

ALGEBRA

1. *Students will be expected to* demonstrate an understanding that equations and inequalities express mathematical relationships and can be used to model physical relationships.

   Students will be expected to:

   1.1 solve and verify the solution of any first degree equation in one variable by applying formal algebraic procedures

      1.1.1 represent problems with equations, and solve problems through the solution of the equations

   1.2 solve formulae for given subjects
   1.3 solve, graph and verify the solution of first degree inequalities in one variable

      1.3.1 represent problems with inequalities, and solve problems through the solution of the inequalities.

2. *Students will be expected to* demonstrate an understanding that polynomials can be written as a product of their factors.

   Students will be expected to:

   2.1 factor (over the integers) trinomials of the form $ax^2 + bx + c$, $a, b, c \blacksquare I$
   2.2 differentiate between quadratic expressions and quadratic equations
   2.3 solve and verify the solution of quadratic equations that can be factored

      2.3.1 use a graphing calculator or graphic computer software to recognize the relationship between the solutions to a quadratic equation and its corresponding graph

   2.4 solve and verify the solution of quadratic equations that can be reduced easily to $x^2 = a$

      2.4.1 use a graphing calculator or graphic computer software to recognize the relationship between the solutions to a quadratic equation and its corresponding graph.

LINEAR RELATIONS

1. *Students will be expected to* demonstrate an understanding that the equations and graphs of linear relations can be determined uniquely from given characteristics of the relation.

   Students will be expected to:

   1.1 write the equation and draw the graph of a linear relation given its slope and the y-intercept
   1.2 write the equation and draw the graph of a linear relation given any two points
   1.3 write equations and draw the graph of a linear relation given a point and a slope
   1.4 write the equation and draw the graph of a linear relation given a point and the equation of a parallel or perpendicular line
   1.5 write the equation and draw the graph of a linear relation given any combination of characteristics that uniquely define it

1.5.1 use calculators or computers to draw graphs of linear relations
1.5.2 solve problems that involve writing the equations or drawing the graphs of linear relations
1.5.3 interpret the equations of linear relations that represent physical phenomena.

SYSTEMS OF EQUATIONS

1. *Students will be expected to* demonstrate an understanding that some mathematical circumstances can be represented algebraically by a system of two equations in two unknowns.

   Students will be expected to:

   1.1 solve systems of linear equations in two unknowns graphically

      1.1.1 use a graphing calculator or graphic computer software to graph two linear equations
      1.1.2 solve problems involving the graphs of two linear equations

   1.2 solve systems of linear equations in two unknowns algebraically

      1.2.1 solve problems that can be represented by a system of two linear equations in two unknowns.

PROBABILITY

1. *Students will be expected to* demonstrate an understanding that probability describes the likelihood of the occurrence of an event and is a number between 0 and 1.

   Students will be expected to:

   1.1 use the language of probability
   1.2 determine the sample space for simple events
   1.3 determine the probability of simple events through experiments

      1.3.1 carry out an investigation to determine the experimental probability of an event

   1.4 determine the theoretical probability of events that have easily countable sample spaces

      1.4.1 compare theoretical probabilities to those for the same event generated empirically.

2. *Students will be expected to* demonstrate an understanding that the probability of a compound event can be determined from the probability of the individual events.

   Students will be expected to:

   2.1 find the probability of two or more events occurring together by the application of the multiplication law for independent events, $P(A \text{ and } B) = P(A) \times P(B)$, and for dependent events, $P(A \text{ and } B) = P(A) \times P(B|A)$

      2.1.1 solve problems that involve finding the probability of the occurrence of two or more events
      2.1.2 determine experimentally the probability of two or more events occurring together, and compare it to the theoretical probability.

GEOMETRY

1. *Students will be expected to* demonstrate an understanding that within closed two-dimensional figures, there are relationships amongst the sides and angles.

   Students will be expected to:

   1.1 determine patterns that can be used to discover relationships pertaining to closed polygons

      1.1.1 determine the sum of the interior angles of any polygon
      1.1.2 determine the relationship between the number of sides and diagonals in a polygon

   1.2 determine patterns to develop a formula for perimeter, which is the distance around a figure or an object

      1.2.1 calculate the perimeter of any regular polygon
      1.2.2 solve problems involving the perimeter of regular polygons
      1.2.3 determine that the ratio between the circumference and diameter of a circle is constant
      1.2.4 solve problems involving the circumference of circles

   1.3 determine patterns to develop a formula for area that is a measure of the number of non-overlapping square units that can be enclosed within a figure

      1.3.1 calculate the area of any regular polygon
      1.3.2 calculate the areas of irregular regions that can be subdivided into regular polygons
      1.3.3 solve problems involving the area of polygons
      1.3.4 determine what figure encloses the largest area for a given perimeter
      1.3.5 calculate the area of a circle.

2. *Students will be expected to* demonstrate an understanding that within three-dimensional figures, there are relationships amongst the sides, edges and vertices.

   Students will be expected to:

   2.1 determine patterns that can be used to discover relationships pertaining to three-dimensional figures

      2.1.1 determine the relationship between the number of edges, vertices and faces in a simple polyhedron (Euler's formula $V - E + F = 2$)
      2.1.2 determine the relationship between the number of faces and diagonals in a polyhedron

   2.2 determine patterns to develop a formula for the surface area of a rectangular solid, cone, sphere, cylinder and pyramid

      2.2.1 calculate the surface area of a rectangular solid, cone, sphere, cylinder and pyramid
      2.2.2 calculate the surface area of irregular solids that can be subdivided
      2.2.3 solve problems involving surface area for regular and irregular solids

   2.3 determine patterns to develop a formula for the volume of regular solids (area of base times height)

      2.3.1 calculate the volume of regular solids (area of base times height)
      2.3.2 calculate the volume of irregular solids that can be subdivided

2.3.3 solve problems involving the volume of regular and irregular solids
2.3.4 determine what figure encloses the greatest volume for a given surface area.

TRIGONOMETRY

1. *Students will be expected to* demonstrate an understanding that two triangles are similar if corresponding angles are equal and the lengths of the corresponding sides are proportional.

   Students will be expected to:

   1.1 identify and express the similarity correspondences between similar triangles
   1.2 use the relationship between the lengths of the sides of similar triangles to calculate the lengths of unknown sides

      1.2.1 derive the properties of similar triangles experimentally
      1.2.2 solve problems involving the use of the properties of similar triangles
      1.2.3 use the properties of similar triangles to investigate the sine, cosine and tangent ratios in similar right triangles.

2. *Students will be expected to* demonstrate an understanding that the sine, cosine and tangent ratios of a given angle are particular ratios between pairs of sides of a right triangle.

   Students will be expected to:

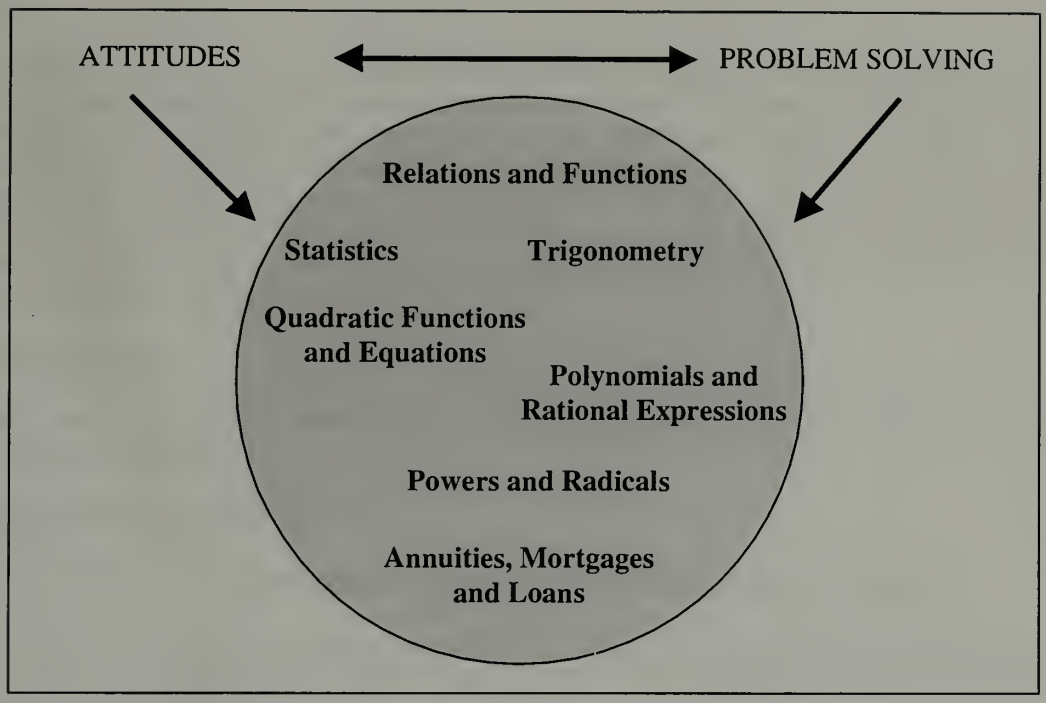
   2.1 find the sine, cosine and tangent ratios, and the measures of the acute angles within right triangles, given the measures of any two sides

      2.1.1 find the sine, cosine and tangent ratios of any acute angle, using a calculator
      2.1.2 find the measure of an acute angle given the value of one of the trigonometric ratios of the angle, using a calculator
      2.1.3 solve problems in which an acute angle measure in a right triangle must be found

   2.2 find the measure of a side of a right triangle given the measure of an acute angle and the length of a side

      2.2.1 solve problems in which the length of a side of a right triangle is unknown.

MATHEMATICS 33 PROGRAM STRUCTURE



POWERS AND RADICALS

Overview

This unit looks at powers and exponents, integers as exponents, approximating square roots, rational exponents, the Pythagorean relation, operation with radicals, using calculators with powers and radicals, solving problems and application. Historically, the reason for studying radicals was to make the estimation of a rational number calculation easier. With the advent of technology the value of performing basic operation radicals should speed up this process.

Learner Expectations

1. *Students will be expected to* demonstrate an understanding that radical expressions and equations can be simplified by performing basic operations on numbers and expressions written in radical form.

   Students will be expected to:

   1.1 change the form of second and third order radicals from mixed to entire and vice versa
   1.2 perform the operations of addition, subtraction, multiplication and division involving second order radicals

      1.2.1 rationalize radicals with monomial denominators involving one second order expression

   1.3 solve second order radical equations

      1.3.1 solve problems involving radical equations.

ANNUITIES, MORTGAGES AND LOANS

Overview

Topics in this unit can be made more relevant and immediate for the students by using items from current newspapers, banks, lending and investment institutions. These items may include advertisements for retirement plans, investments, bonds, mortgages, savings plans and annuities. Students will learn to differentiate between mortgages and loan payments. Using tables, as opposed to calculating annuities as a geometric series, students will learn different aspects of financing. This section provides a real-life experience using mathematics. Through problem solving (e.g., how the repayment schedule on loans and mortgages works; the effects on the amount borrowed, interest rates, number of payments over a certain period), the student can attain basic skills.

Learner Expectations

1. *Students will be expected to* demonstrate an understanding that an annuity is a sequence of payments paid in regular intervals.

   Students will be expected to:

   1.1 calculate the amount and present value of an annuity, using tables or calculators

      1.1.1 solve simple problems involving the amount and present value of an annuity
      1.2.2 use a calculator or computer spreadsheet to help calculate the amount and present value of an annuity

2. *Students will be expected to* demonstrate an understanding that the repayment schedule on loans and mortgages is determined by the amount borrowed, the interest rate, the number of payments and the total time over which repayment occurs.

   Students will be expected to:

   2.1 differentiate between a mortgage and other types of loans
   2.2 calculate mortgage and loan payments, using tables

      2.2.1 solve problems involving the calculation of payments on loans and mortgages
      2.2.2 use a calculator or computer to help in the solution of problems involving loans and mortgages.

STATISTICS

Overview

Statistics forms a very essential part of the modern mathematics curriculum. The daily barrage of information must not only be understood but also used by students. Statistics includes all the background thinking that occurs long before any work is done.

The techniques and skills used include bivariate data on a scatter plot, strength and direction of the correlation between two variables. The line and equations can be used to make predictions. Surveys are an integral part of inferential statistics and students must understand that the results of surveys can be interpreted with measurable degrees of confidence. The knowledge of this type of mathematics will help students become intelligent consumers who can make critical and informed decisions.

Learner Expectations

1. *Students will be expected to* demonstrate an understanding that a bivariate distribution involves two variables that may have some relationship to each other.

   Students will be expected to:

   1.1 plot sets of bivariate data on a scatter plot
   1.2 plot a line of best fit on a scatter plot, using the median fit method
   1.3 develop and use prediction equations of the line of best fit to make inferences for populations
   1.4 determine the apparent correlation between the variables of a bivariate distribution from a scatter plot, including the strength and direction of the correlation
   1.5 collect, organize and analyze sets of bivariate data

      1.5.1 apply statistical processes and statistical reasoning in **investigations** involving bivariate data.

2. *Students will be expected to* demonstrate an understanding that the results of a survey can be interpreted with measurable degrees of confidence.

   Students will be expected to:

   2.1 distinguish between a population and a sample and assess the strengths, weaknesses and biases of given samples
   2.2 collect and organize the results of yes/no surveys taken from defined samples

      2.2.1 design and administer a simple survey
      2.2.2 collect and organize the results of a simple survey

   2.3 draw box plots of the results of multiple samples (50 per cent and 90 per cent box plots)

      2.3.1 carry out investigations involving multiple samples taken from populations with known and unknown proportions of yes responses

   2.4 use charts of 90 per cent box plots to find the confidence interval within which a survey result can be interpreted

      2.4.1 use statistical inferences to solve problems based on 90 per cent confidence intervals

   2.5 draw statistical conclusions, make inferences to populations and explain the confidence with which such conclusions and inferences are made based on the results of yes/no surveys

      2.5.1 design and administer a survey to a random sample of a population, collect and organize the responses, and analyze the results, including making inferences to the population and evaluating the results for the confidence with which they may be held.

TRIGONOMETRY

Overview

The main focus of this unit is the property and theorems related to trigonometry. The study of trigonometry builds on the understanding of the trigonometric ratios and how to use these ratios to solve problems, and on the study of functions. The ratios of right-angle trigonometry are generalized to both trigonometric and circular functions. Understanding of these functions and the connections between

geometry and algebra are important in the future development of such topics. Students are able to see the relationship between right-angle triangles and the oblique type of triangle, through the development of the sine and cosine law. The final section offers many practical applications requiring students to use the Law of Sines and Cosines.

Learner Expectations

1. *Students will be expected to* demonstrate an understanding that angles can be drawn on a coordinate plane and that their trigonometric ratios can be determined with respect to the coordinates of points on the plane.

   Students will be expected to:

   1.1 recognize and sketch positive and negative angles in standard position on a coordinate plane

      1.1.1 define and determine principle and co-terminal angles in a coordinate plane

   1.2 determine the reference angle for an angle drawn in standard position on a coordinate plane
   1.3 express the trigonometric ratios for any angle, θ, in standard position on a coordinate plane in terms of x, y and r, where r is the length of the terminal side and (x, y) the coordinates of the end-point
   1.4 determine the sine, cosine and tangent ratios for any angle

      1.4.1 use a calculator to determine the sine, cosine and tangent ratios of any angle

   1.5 determine any two values of x, y, r and θ given the other two
   1.6 use a graphing calculator or computer to investigate the graphs of the sine curve

      1.6.1 use a graphing calculator or computer to investigate the effects of the parameter a, b, of the function $y = a \sin bx$.

2. *Students will be expected to* demonstrate an understanding that the methods used in the solution of right triangles can be used to develop laws for use in the solution of oblique triangles.

   Students will be expected to:

   2.1 find the measures of sides and angles in diagrams involving multiple right triangles in two or three dimensions

      2.1.1 solve problems involving multiple right triangles in two or three dimensions

   2.2 find the measures of unknown sides and angles in oblique triangles by applying the sine law

      2.2.1 illustrate the sine law, using examples
      2.2.2 solve problems, using the ambiguous case of the sine law
      2.2.3 illustrate the ambiguous case of the sine law

   2.3 find the measures of unknown sides and angles in oblique triangles by applying the cosine law

      2.3.1 recognize that the cosine law is the generalization of the Pythagorean theorem
      2.3.2 illustrate the cosine law, using examples
      2.3.3 solve problems, using the cosine law.

POLYNOMIALS AND RATIONAL EXPRESSIONS

Overview

In this unit, the student will be exposed to rational expressions (algebraic fractions) of polynomials. The student will explore non-permissible values of rational expression; what happens to the expression when a certain value is substituted for the denominator. Factoring will play a very important role in simplifying these rational expressions. Operations with addition, subtraction, multiplication and dimension can be performed on such expressions. Solving problems involving equations containing rational expressions will help the students understand their application.

Learner Expectations

1. *Students will be expected to* demonstrate an understanding that rational expressions are fractions in which the numerators and denominators are polynomials and that operations can be performed on such expressions.

   Students will be expected to:

   1.1 determine the non-permissible values in rational expressions in one variable

      1.1.1 use a graphing calculator and/or computers to graph rational functions to determine non-permissible values

   1.2 simplify rational expressions by factoring
   1.3 perform the operations of multiplication and division on rational expressions
   1.4 perform the operations of addition and subtraction on rational expressions
   1.5 solve equations involving rational expressions

      1.5.1 solve problems involving equations containing rational expressions.

RELATIONS AND FUNCTIONS

Overview

This unit introduces the concept of relations and functions with an emphasis on graphical representation. Interpreting and graphing relations are reviewed. Linear and non-linear relations are illustrated through graphical representations, and their domain and range are determined.

Functions are introduced as special relations that satisfy certain conditions (e.g., vertical line test). Functional notation and evaluating functions using functional notation are discussed. The inverse of a function is determined in the same way as the universe of a relation. The final section of this unit deals with the various transformations of functions from the form $y = f(x)$ to $y = cf(x - a) + b$.

Learner Expectations

1. *Students will be expected to* demonstrate an understanding that certain observed real-world phenomena are quantitatively related and that these relations can be described graphically, with sets of ordered pairs, rules and equations.

   Students will be expected to:

   1.1 graph relations that describe physical phenomena or everyday situations

1.1.1 solve problems by graphing and interpreting the graphs that describe physical phenomena and everyday occurrences

1.2 determine the domain and range of the relations from graphs and sets of ordered pairs.

2. *Students will be expected to* demonstrate an understanding that for some relations, called functions, the value of the independent variable (domain) uniquely determines the value of the dependent variable (value of the function, range).

Students will be expected to:

2.1 represent simple mathematical situations such as direct and inverse variations with tables of values, identify the dependent and independent variables, and express the domain and range, appropriately noting any restrictions
2.2 interpolate and extrapolate from the graphs of simple functional relationships
2.3 use functional notation and graphs to describe simple relationships and evaluate them by substituting appropriate replacements of the domain

2.3.1 solve problems algebraically or by using and interpreting graphs that represent functions

2.4 illustrate and recognize different kinds of functions algebraically and graphically from the following list: linear functions, polynomial functions (including quadratic and cubic functions), reciprocal functions, absolute value functions and exponential functions

2.4.1 draw and analyze the graphs of functions using calculators and computers
2.4.2 analyze the graphs of the inverse of functions (e.g., linear, quadratic, reciprocals).

3. *Students will be expected to* demonstrate an understanding of how particular parameters can be used to effect translations, reflections or vertical stretchings of any function.

Students will be expected to:

3.1 describe the transformation effects on the graph of $y = f(x)$ of the parameters a and b in $y = f(x - a) + b$

3.1.1 investigate the effects of the parameters a and b on the graph of $y = f(x - a) + b$

3.2 describe the transformation effects on the graph of $y = f(x)$ of the parameter c in $y = cf(x)$

3.2.1 investigate the effects of the parameter c on the graph of $y = cf(x)$

3.3 describe and sketch the graphs of $y = cf(x - a) + b$

3.3.1 use a graphing calculator and/or computers to graph functions in the form $y = cf(x - a) + b$.

QUADRATIC FUNCTIONS AND EQUATIONS

Overview

This unit analyzes quadratic functions with respect to graphs and solving quadratic equations with graphs. The graphs are used to determine the domain and range, the maximum or minimum points, the axis of symmetry, x- and y-intercepts, and the direction of opening of quadratic functions. The equation

form of the quadratic function is converted from the general form $y = ax^2 + bx + c$ to the more convenient standard form $y = a(x - h)^2 + k$.

Quadratic equations will be solved using various methods, such as factoring, completing the square and the development of the quadratic formula. The unit will discuss solving problems that involve equations with radicals and rational expressions whose solutions are the roots of quadratic equations.

Learner Expectations

1. *Students will be expected to* demonstrate an understanding that any function that can be written in the form $y = ax^2 + bx + c$, $a \neq 0$, is a quadratic function and that individual quadratic functions have unique characteristics and graphs.

   Students will be expected to:

   1.1 sketch the graphs of quadratic functions written in standard form, $y = a(x - h)^2 + k$

      1.1.1 investigate the effects of the parameters a, h and k in $y = a(x - h)^2 + k$ using a graphing calculator or computer

   1.2 transform quadratic functions from the general form $y = ax^2 + bx + c$ to the standard form $y = a(x - h)^2 + k$ by completing the square

      1.2.1 find the vertex, axis of symmetry, domain, range, maximum or minimum values and y-intercepts of a quadratic function
      1.2.2 solve problems that involve quadratic functions, by analyzing the functions in graphical or equation form.

2. *Students will be expected to* demonstrate an understanding that finding the x-intercepts of the quadratic function $y = ax^2 + bx + c$ is equivalent to solving the quadratic equation $ax^2 + bx + c = 0$.

   Students will be expected to:

   2.1 use the quadratic formula to solve quadratic equations

      2.1.1 determine the x-intercepts of the quadratic function
      2.1.2 solve problems that involve quadratic equations

   2.2 recognize and explain the relationship between the x-intercepts of the quadratic functions and the roots of their corresponding equations
   2.3 solve equations with radicals and rational expressions whose solutions are the roots of quadratic equations

      2.3.1 solve problems that involve equations with radicals and rational expressions whose solutions are the roots of quadratic equations.

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